## INTEGRATION TECHNIQUES

This review of integration techniques is in no way complete. It is vital for your success that you attempt a large number of problems from the text (even more than are assigned). There is no substitute for practice and experience. I hope that this guide helps you organize your studying.

On page 495 of the text you can see a table of the integrals we can do in one step. Really, the integrals from this table that I want you to assume are doable in one step are 1-14 and 17. Those are the ones you can assume. If your integral is not one of those, then you need some simplifying method. The first thing you should do is look for any possible substitutions or algebraic simplifications. Then you should try one of our four new methods. These methods, and when to choose them, are illustrated below:


Products, log's, inverse trig

## INTEGRATION BY

## PARTS

$$
\begin{array}{rrr}
\mathrm{u}= & \mathrm{dv}= \\
\mathrm{du}= & \mathrm{v}=
\end{array}
$$

If you're stuck on choosing $u$ remember LIPET. (But after you get comfortable with this method, you shouldn't need LIPET anymore)
sin's, cos's, tan's, sec's

## TRIG. INTEGRALS

1. Odd $\cos \rightarrow u=\sin (x)$
2. Odd $\sin \rightarrow u=\cos (x)$
3. Even $\sec \rightarrow u=\tan (x)$
4. Odd $\tan \rightarrow u=\sec (x)$
5. Even $\sin \& \cos$
$\rightarrow$ Half Angle Identities
In the first 4 cases you need the identities:

$$
\begin{aligned}
& \sin ^{2}(x)=1-\cos ^{2}(x) \\
& \cos ^{2}(x)=1-\sin ^{2}(x) \\
& \tan ^{2}(x)=\sec ^{2}(x)-1 \\
& \sec ^{2}(x)=\tan ^{2}(x)+1
\end{aligned}
$$

For the $5^{\text {th }}$ case, you need the half angle identities: $\sin ^{2}(x)=(1-\cos (2 x)) / 2$ $\cos ^{2}(x)=(1+\cos (2 x)) / 2$ $\sin (\mathrm{x}) \cos (\mathrm{x})=\sin (2 \mathrm{x}) / 2$

| $a^{2}-x^{2}, x^{2}+a^{2}, x^{2}-a^{2}$ |
| :---: |
| or if quadratic doesn't factor |

## TRIG. SUBSTITUTION

If the quadratic has a linear term ( 'middle term') and it doesn't factor, then you need to complete the square. ( $1 / 2$ of middle term, square, add and subtract value)
The rest of the method follows by making the correct substitution.

$$
\begin{aligned}
& x=a \sin (\theta) \\
& x=a \tan (\theta) \\
& x=a \sec (\theta)
\end{aligned}
$$

At the end, draw and label the TRIANGLE to get back to x's.
rational functions where the bottom factors

## PARTIAL FRACTIONS

Divide if the power of top is bigger than power on bottom.

Then factor the bottom and set up and solve the partial fraction decomposition.

Distinct Linear Factors $\rightarrow$ Determine a constant for each factor.

Non-Distinct Linear Factors
$\rightarrow$ Determine a constant for each factor, along with each power from 1 up to the number of times repeated.

Irreducible Quadratic Factor
$\rightarrow$ Complete the square, the numerator of the factor is $A x+B$.

